Prim’s algorithm ...
Minimum Spanning tree (MST)

• Given an undirected connected graph $G$
  – The edges are labelled by weight
• Spanning tree of $G$
  – a tree containing all vertices in $G$
• Minimum spanning tree
  – a spanning tree of $G$ with minimum weight
Examples

Graph $G$
(edge label is weight)

Spanning trees of $G$

MST
Algorithm

ALGORITHM Prim(G)

//Prim’s algorithm for constructing a minimum spanning tree
//Input: A weighted connected graph G = <V,E>
//Output: E_T, the set of edges composing a minimum spanning tree of G

\[ V_T \leftarrow \{v_0\} \]
\[ E_T \leftarrow \emptyset \]

for \( i \leftarrow 1 \) to \( |V| - 1 \) do

find minimum weight edge \( e^*=(v^*, u^*) \) among all the edges \((v,u)\) such that \( v \) is in \( V_T \) and \( u \) is in \( V - V_T \)

\[ V_T \leftarrow V_T \cup \{u^*\} \]
\[ E_T \leftarrow E_T \cup \{e^*\} \]

return \( E_T \)
Prim’s algorithm
Prim’s algorithm

Tree vertices
- a
- b
- c
- d
- e

Remaining vertices
- b(a, 3)
- c(-, \infty)
- d(-, \infty)
- e(a, 6)
- f(a, 5)

Illustration

\begin{tikzpicture}
\node (a) at (0,0) {a};
\node (b) at (2,2) {b};
\node (c) at (4,2) {c};
\node (d) at (6,0) {d};
\node (e) at (2,-2) {e};
\node (f) at (4,-2) {f};

\draw[red] (a) -- (b) node [midway] {3};
\draw (b) -- (c) node [midway] {1};
\draw (c) -- (d) node [midway] {6};
\draw (d) -- (e) node [midway] {8};
\draw (e) -- (f) node [midway] {4};
\draw (f) -- (a) node [midway] {5};
\end{tikzpicture}
Prim’s algorithm

Tree vertices
b(a,3)

c(b,1)
e(a,6)

Remaining vertices
d(-,∞)
f(b,4)

Illustration
Prim’s algorithm

Tree vertices: c(b,1)
Remaining vertices: d(c,6), e(a,6), f(b,4)

Illustration:
Prim’s algorithm

Tree vertices Remaining vertices
f(b,4) d(f,5) e(f,2)
Prim’s algorithm

Tree vertices Remaining vertices
e(f,2) d(f,5)